

## MODELLING CHAOTIC VIBRATIONS USING NASTRAN

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## ABSTRACT:

Due to the unavailability and, later, prohibitive cost of the computational power required, many phenomena in nonlinear dynamic systems have in the past been addressed in terms of linear systems. Linear systems respond to periodic inputs with periodic outputs, and may be characterised in the time domain or in the frequency domain as convenient. Reduction to the frequency domain is frequently desirable to reduce the amount of computation required for solution.

Nonlinear systems are only soluble in the time domain, and may exhibit a time history which is extremely sensitive to initial conditions. Such systems are termed chaotic.

Dynamic buckling, aeroelasticity, fatigue analysis, control systems and electromechanical actuators are among the areas where chaotic vibrations have been observed. Direct transient analysis over a long time period presents a ready means of simulating the behaviour of self-excited or externally excited nonlinear systems for a range of experimental parameters, either to characterize chaotic behaviour for development of load spectra, or to define its envelope and preclude its occurrence.

## INTRODUCTION:

Chaotic systems have been defined as those whose time history is highly dependent on initial conditions. Without coining the term "chaos", Henri Poincare (1) informally stated precisely this definition early in the century, and there can be little doubt that earlier than this the concept was known to dynamicists, and remained undeveloped because, in the absence of digital computers and modern instrumentation, it was not a profitable field of inquiry.

The availability of computational power at an unprecedentedly low cost has extended the range of chaotic phenomena in mechanical systems which may profitably be investigated. Such investigation requires solution of the equations of motion of the system in the time domain over a long time period and the subsequent processing of the large body of data acquired to produce phase plots, power spectral densities, peak loads etc. In effect the computer is used to simulate in the time domain a physical test in the time domain (such as a shaker table test for vibration, a wind tunnel test for aeroelasticity, or the experimental observation of the behaviour of an electromechanical system under periodic actuation). Results from the simulation may be processed in the same manner as data from physical experimentation, to produce power spectral densities, Poincare plots and other means of providing insight into the system's behaviour. Extension of analysis beyond the linear domain has the potential of allowing less conservative design assumptions, and of providing an alternative, less statistically oriented approach to load spectrum development and fatigue analysis.

## CHAOTIC VIBRATIONS:

Consider a linear dynamic system subject to a periodic input. The response of the system to this input at all degrees of freedom will be a periodic output of amplitude and phase shift dependent upon the mass, stiffness and damping of the system.

The system can be defined equivalently either by equations for displacement as a function of time, or by equations for amplitude and phase of displacement for different input frequencies and amplitudes. The direct response and random analysis disciplines within NASTRAN use the latter approach to generate an output Power Spectral Density (PSD) for a given input PSD to a linear system. Significant modes are determined by modal analysis, after which the amplitude and phase of the system's response to excitation at and around these frequencies using the direct response method. Finally, an input PSD is applied to the data from the direct response analysis to produce an output PSD. of displacement, load, stress or whatever variable is required.

The results obtained are statistical in nature, providing a non-zero value of spectral density for any amplitude. The analyst must determine an amplitude at which nonlinear factors will truncate the PSD curve. This level is somewhat variable, and is generally taken to be between 3 and 10 times the RMS value. Selection of an appropriate truncation point can present problems to the analyst.

Introduction of significantly nonlinear spring constants or nonuniform damping requires that the system must be analysed, in NASTRAN, by direct time integration. Depending upon the degree of nonlinearity and the degree of damping the response to a periodic input may be periodic, quasiperiodic, limit cycle or chaotic. Despite the distinction in names, the first three categories are all periodic in the sense that they may be described by a Fourier series of finite length.

A quasiperiodic system differs from a periodic one in that, although it is expressible as a series of finite length, the frequency components are cannot be expressed as a rational number. It appears, therefore, that quasiperiodic oscillations can not be modelled numerically. Numerical approximation will reduce a quasiperiodic motion to a low frequency periodic one.

Limit cycle vibration is self-excited vibration whose amplitude is limited by non-linear effects. Classical flutter is an example of limit cycle vibration.

Classical flutter theory is limited to the location of regions of negative damping in a linear aeroelastic model, with the purpose of ensuring that these regions are outside the flight envelope. A time-domain solution of nonlinear aeroelastic equations offers the prospect of defining the amplitude of an oscillation which may in reality be either limit cycle or chaotic.

A chaotic system, subject to self-excitation or to a periodic input, will produce a non-periodic output. The system is entirely deterministic and, given the displacement, velocity and acceleration of all degrees of freedom at time  $t_1$ , the same variables may be calculated at any future time  $t_2$ . It is interesting to note that the process can not necessarily be reversed to find the state of the system at any prior time. It follows from the above that, if the system is sampled at a rate equal to the period of the input excitation, with any phase shift, the same system state will never recur, since if it did the system would thereafter behave periodically. A self-excited system, not being subject to a periodic external load,

will never exhibit the same state at any sampling frequency.

A useful definition of chaotic vibration might be a response to a periodic input which can not be characterised by a Fourier series of finite length.

Time-domain analysis of potentially chaotic vibrations subject to periodic excitation provides information as to range of frequencies and amplitudes of excitation for which a non-periodic response may be expected, by examination of power spectral density and Poincare plots, and also information allowing an informed decision as to where to truncate the output PSD from a random response analysis, if the response should prove to be approximately linear for the levels of excitation of interest. For systems where the excitation is dominated by a relatively small number of frequencies, the system can be solved directly over a suitable time period by using a combination of dynamic load cards to provide excitation with several frequency components. Input excitations associated with rotating machinery are a case in point.

In self-excited oscillations, such as flutter, a non-linear analysis in the time domain can, by accounting for geometric and material nonlinearities, provide the limit amplitude of a periodic oscillation, or an envelope for chaotic oscillation. Other potentially chaotic self-excited systems include control systems with hysteresis and "galloping" of cables.

In all these cases it is potentially of interest to determine whether the oscillation will result in immediate catastrophic failure, will produce stresses affecting the life of the structure or will be limited at a benign level by nonlinearities.

#### ATTRACTORS, POINCARÉ MAPS AND POWER SPECTRAL DENSITY

Given a time history of a time-domain NASTRAN transient analysis, of a self- or periodically excited system, the generation of an output PSD is an obvious and simple step. This involves operating on the output data in precisely the same manner as would be done with experimental data. At least as important for potentially chaotic systems are phase plots and Poincare plots, where the variable of interest (usually position) is the ordinate and its first derivative is the abscissa.

For a periodic oscillation, either externally or self-excited, such a plot will form a closed path. The simplest case, an undamped single DOF oscillator, appears in a phase plot as an ellipse (or a circle if appropriately scaled) centered on the equilibrium position of the oscillator. The addition of a damping term will produce a plot in phase space which spirals in to the equilibrium position. The equilibrium point is an attractor for the single DOF damped spring, since as the initial disturbance of the system dies away, the system tends to this state. For a periodic oscillation not decaying to equilibrium, such as the undamped single DOF oscillator, the attractor is a closed curve. Sampling at a rate equal to the natural frequency will reduce the plot to a single point. Such plots in phase space are termed Poincare plots. More complex periodic oscillations, having several frequency components due to a forcing function with several frequencies, will appear in the phase plot as interleaved curves. By selecting an appropriate sampling rate the output data will be a finite number of loci defining a closed curve, with data points repeating after a finite number of cycles. In a single DOF system, sampled at the forcing function frequency, the coincidence of displacement and velocity implies a coincidence of acceleration, and consequently the curve in the phase plot can not intersect itself.

For a quasiperiodic oscillation the attractor will form a closed curve sampled at in phase space. Although all points will lie on the curve, none

Results of the analysis may be interpreted in the same way as those of a physical test.

(1): A time history of displacement or velocity may exhibit a clear periodicity or may not. In the latter case the cause could be either chaotic motion or the combination of several periodic components.

(2): Power Spectral Density Analysis of the system response to a single frequency forcing function. A system verging upon chaos will exhibit several harmonics of the driving frequency, with the response becoming broad-band as the system enters the chaotic regime. Judgement as to the presence or absence of chaos must be made with regard to the system analysed. In the case analysed below a single DOF system produces several harmonics for certain levels of periodic excitation. The conclusions drawn from it would not necessarily be justified from observations of a single node in a complex structure.

(3,4,5): Phase plane observation, Poincare and 3-D plots: These are discussed in some detail above.

#### DYNAMIC MODEL OF AN ELECTROMECHANICAL ACTUATOR SYSTEM:

The electromechanical actuator is a known, simple example of a chaotic oscillator, described by Hendricks in 1983 (2).

Fig.(1) shows an electromechanical actuator system wherein the armature is subject to an externally applied dynamic load by application of an electrical current to a coil. Such systems are used in impact print mechanisms, high speed relays and elsewhere.

The system is modelled as an armature GRID with a single DOF moving between two GRIDS each occupying a deep potential well defined by NOLIN1 cards and representing the stops limiting the armature's travel. EPOINT NOLIN1 and TF cards are used to model the impacting of the armature on the stops.

The armature GRID is also attached to ground by a scalar spring whose stiffness was varied during the investigation. The armature thus tends to a rest position with the scalar spring in an unloaded state as shown in Fig.(2).

Also in Fig.(1) is a mechanical fastener transferring load between two components having oversized holes. This system, representative of structural details in aircraft construction or modification, is from a mathematical point of view identical with the actuator system. Note that by applying excitation at one of the constraining grids the same model can represent, without further modification, a system with nonlinear stiffness mounted on a shaker table.

The actuator modelled was given travel between stops of 0.008 inch, peak applied force of 0.8 # and cycle time of 1KHz. These times were based upon an actual device for which data was available and were varied in the course of the study to induce chaotic behaviour. It was determined that a time step of 0.5 uS was required to adequately model the behaviour of the armature and stops during impact. Inspection of the motion of the stops shows that they are restored to equilibrium position between impacts and hence act merely as nonlinear restoring forces on the armature. The armature therefore acted, in effect, as a single DOF nonlinear system. A means of applying a load as a function of space and time was also devised and is described in appendix (1).

#### RESULTS:

##### (1): VARIATION OF DYNAMIC LOAD

Curves of displacement vs. time are plotted in Figs. (3-7) for excitation at 1kHz with peak forces from 0.1 # to 1.2 #, with a travel of 0.008 inch between stops and a weak spring defining the rest position of the armature. It is seen that for the extreme limits of applied load the results do not appear to be periodic.

will be coincident since the ratio of the component frequencies is not a rational number. In a time-domain simulation the distinction from a periodic oscillation is of no importance.

A chaotic oscillation, sampled in this manner, will never repeat itself and may exhibit an interleaved phase plot. This state, not conforming to any of the three cases in classical dynamics, is termed a strange attractor. While the static, periodic and quasiperiodic attractors define closed paths, strange attractors, while being confined to a finite area of phase space, exhibit fine structure within their domain. Alternatively, in lightly damped systems, the plot may appear to be randomly distributed. Such systems are sometimes described as stochastic in nature.

A plot of displacement, velocity and acceleration is of interest. In a self-excited single DOF system, the coincidence of position and velocity imply a coincidence of acceleration, since the acceleration is defined in terms of the other two variables. In chaotic systems, the converse is true and no two points may be coincident in such a plot. In a system with several degrees of freedom the presence or absence of periodicity must be determined by examining, and seeking a coincidence in, the displacement and velocity of all degrees of freedom simultaneously. Graphically, this requires plotting in a space of  $2N$  dimensions where  $N$  is the number of degrees of freedom. For a system subject to an external forcing function, the sampling must be done at the frequency of the forcing function. Given that the analysis must be based upon a simulation of finite span, it will not be possible to prove explicitly that a system is chaotic and only in some clear-cut cases will it be possible to prove the converse.

In practice, as in actual physical testing, several tests may be applied which with a high degree of confidence discriminate between chaotic and periodic behaviour. The envelope defined for motion of an apparently chaotic system is no less useful if the system in fact is periodic with a very long wavelength.

#### APPLICATION OF NASTRAN TRANSIENT ANALYSIS

The paradigm of chaos, the Lorenz attractor, was initially attributed by some to the process of numerical simulation rather than to an underlying physical reality. This proposition will be sympathetically viewed by any analyst who has used NASTRAN to model intermittent contact problems.

In impact studies and similar applications the greatest care must be taken to ensure that the time step is sufficiently small to prevent a node from penetrating a significant distance into a region of high stiffness before the stiffness matrix is updated to reflect this. The effect of such an excessive time step can be that the node is reflected from the collision with a velocity many times that of impact. At the same time, the total number of time steps must, as far as possible, be minimized. For problems such as a single impact, where the regions requiring small timesteps can be estimated, or derived from a preliminary analysis, the problem may be addressed by using several timesteps, with the small ones limited to the appropriate times. In analysis of a chaotic system, however, a large number of cycles must be analysed, and the behaviour is by definition nonperiodic and unpredictable. A single value of time step must be employed and experimentation is required to determine the maximum timestep commensurate with conservation of energy in the system.

The application of periodic dynamic loads required the input of a large amount of data, defining each of many cycles explicitly. This is conveniently done using an external preprocessor to generate the appropriate cards. The numerical and graphical output from direct transient analysis consists of the values of variables as a function of time, as would be the case for a physical test. The desired output of phase plots (velocity vs. position) and power spectral density may be readily obtained, however, from a punch file of the results, either by use of a batch program or by importation into a spreadsheet or database program.

The velocity plots in Figs. (8-12) provide a clearer picture of the armature's behaviour, with almost constant peak velocity for dynamic loads from 0.24 # to 0.8 # and considerable variation outside that envelope. Figs. (13-17) are phase plots of velocity vs. displacement for the same data. Figs. (18-19) show the superimposed plots in the vicinity of the front and back stops. The larger scale reveals considerable fine structure in the curves for peak dynamic loads from 0.24 # to 0.8 #.

Fig.(20) shows the displacement PSD for peak dynamic loads from 0.24 # to 1.2 #. The largest peak is for the 0.4# peak force, but, if the results are normalised for the amplitude of the input force, the 0.24# case will have almost the same magnitude, but with much less marked secondary peaks.

Fig.(21) shows the 0.4# and 0.24# Poincare plots for a sampling rate twice the excitation rate, phase shifted to encompass maximum deflection. The loci near the equilibrium attractor are virtually coincident while the loci near maximum displacement show considerable variation in velocity, but not position. Fig.(22) shows loci for peak forces of 1.2# and 0.24# for a sampling rate equal to the excitation rate. The 0.24# case suggests a long-period periodicity while the 1.2 # case suggests chaotic vibration. Figs. (23-24) are 3-D plots for peak loads of 0.8 # and 1.2 # respectively.

## (2): VARIATION OF NONLINEARITY

By increase of the linear spring constant constraining the armature from 1.0# to 100.0 # it becomes significant with respect to the nonlinear forces. Fig. (25) shows the displacement vs. time for a peak dynamic load of 0.8 # for spring constants of 1.0 and 100.0 respectively. It is apparent from this and the Poincare plot in Fig. (26) that the effect of reducing the range of stiffness is to reduce the tendency to chaos.

## (3): VARIATION ON INPUT FREQUENCY

The effect of increasing input frequency is to increase the tendency to chaos. Fig.(27) shows phase plots for 0.8# peak input force at 1.0, 1.5 and 2.0 Khz. The chaotic behavior at 2.0 KHz is in accordance with test data indicating a maximum stable drive frequency around 1.7 kHz.

## CONCLUSIONS:

The data described above for a magnetomechanical actuator are in agreement with several years of experience in the design, analysis and characterization of such devices. With small modifications, a similar model could be applied to mechanical fasteners in aircraft structures, vibration isolation and other areas where load transmission between pieces of structure is via a nonlinear path. Application of appropriate position-dependent loads should allow nonlinear modelling of flutter and other self-excited phenomena.

Considerable care must be taken to ensure that effects observed are due to physical characteristics of the system and not artifacts of the simulation. Spurious self-excitation of the system due to an inadequate time step is an obvious possibility.

Implementation of automatic time-step variation, such as is available in some other FEA codes, is probably not desirable for an application where there is a significant risk of mistaking numerical artifacts for physical behaviour. A means of specifying a large number of periodic loads on a single card would, however, be desirable.

**REFERENCES:**

- (1): H. Poincare, *The Foundation of Science: Science and Method* (1921)
- (2): F. Hendricks, *Bounce and Chaotic Motion in Print Hammers*, *IBM J Res. Dev.*, 27(3), 273-280

Fig. (1): 1-D Nonlinear System

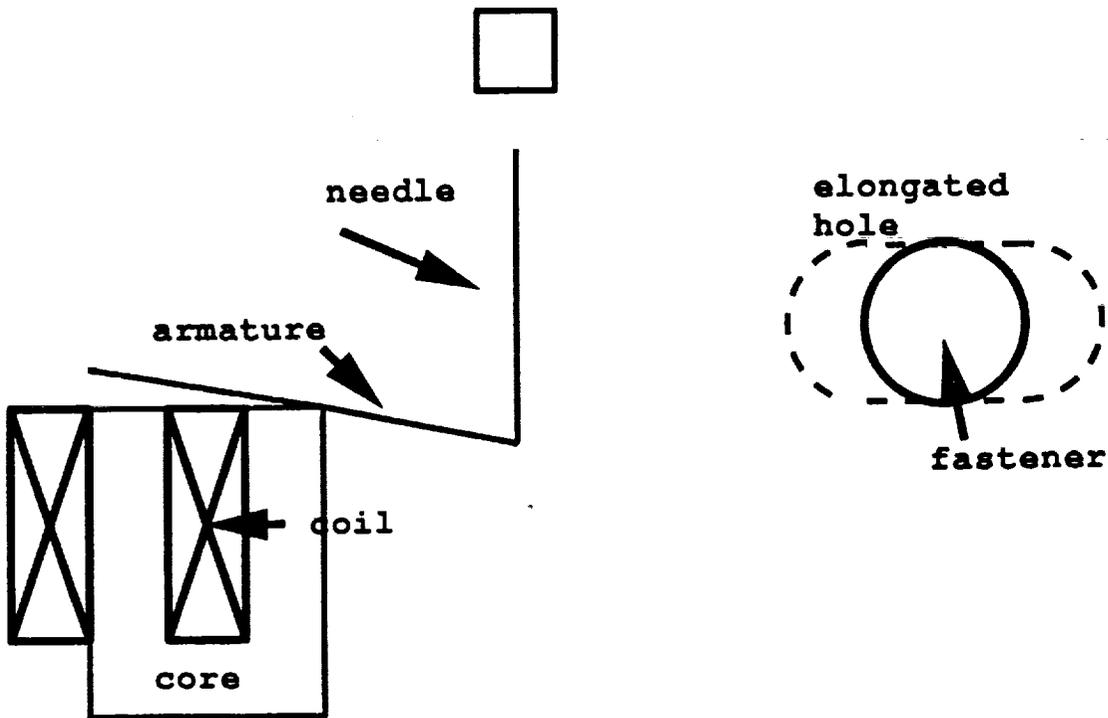
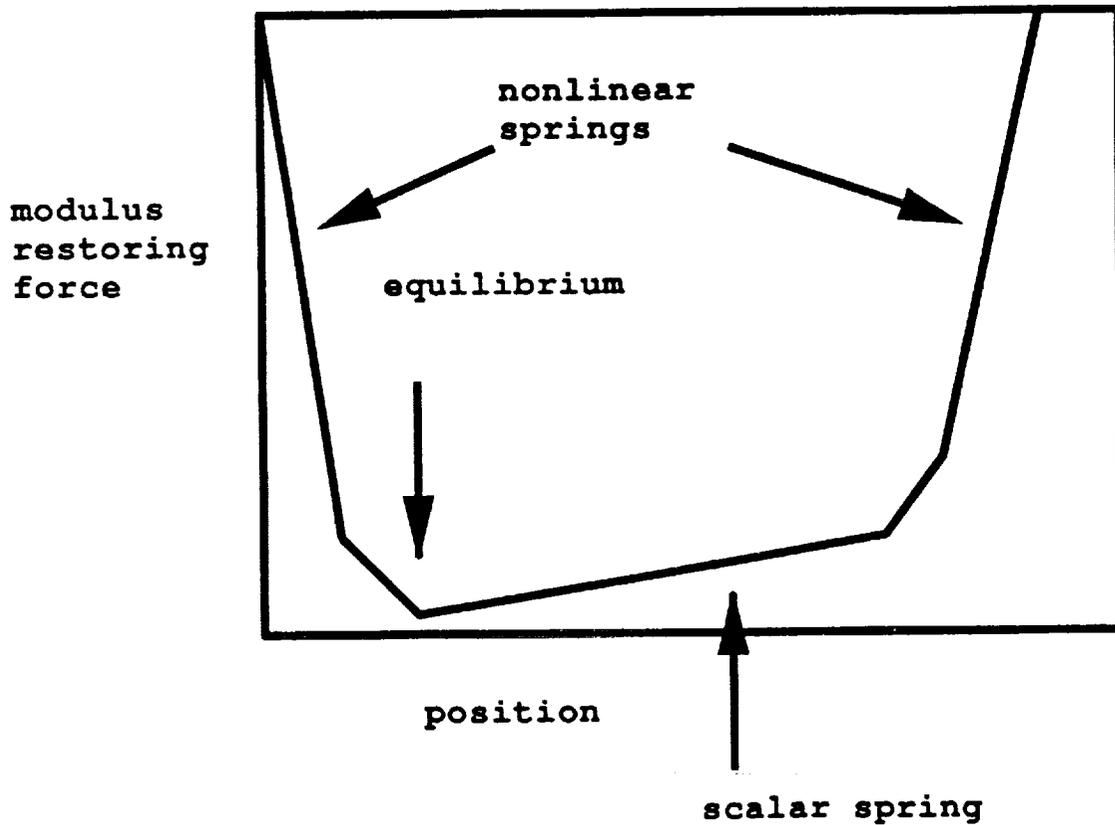
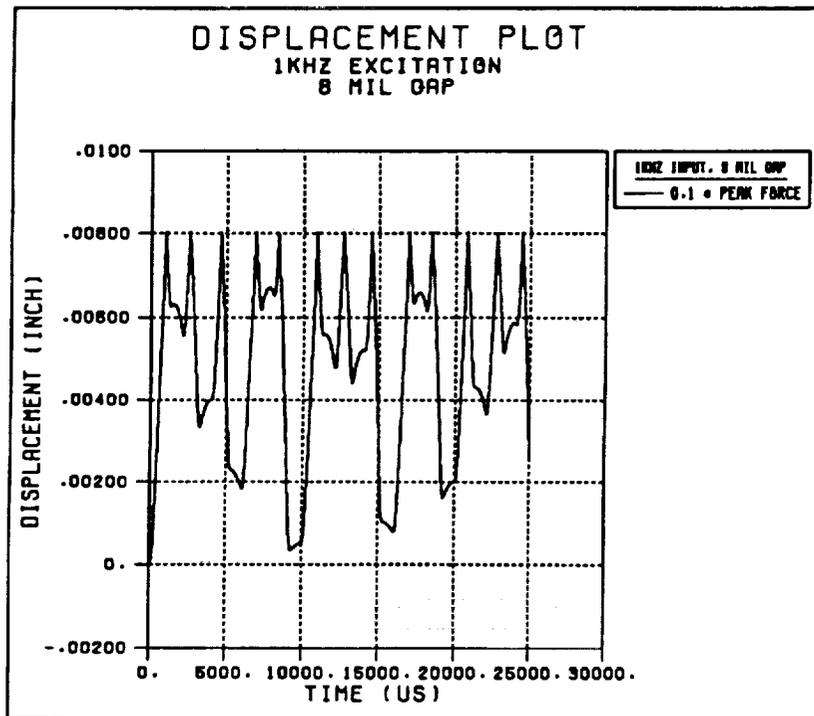
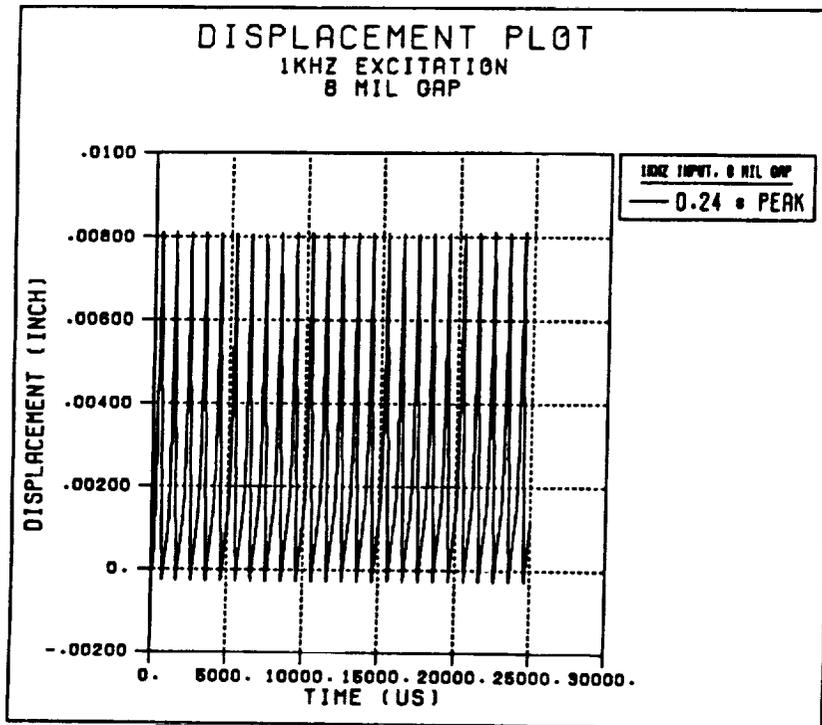
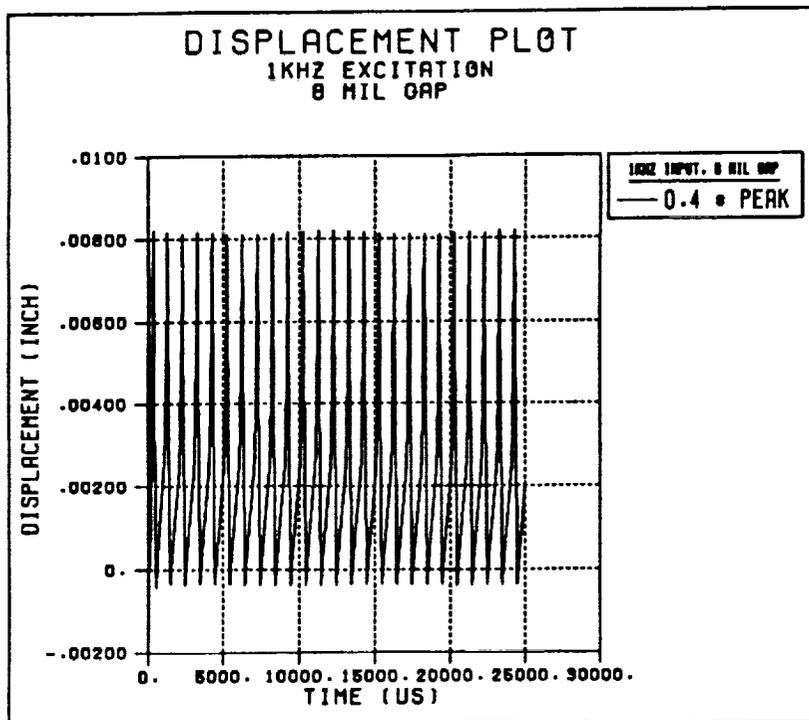
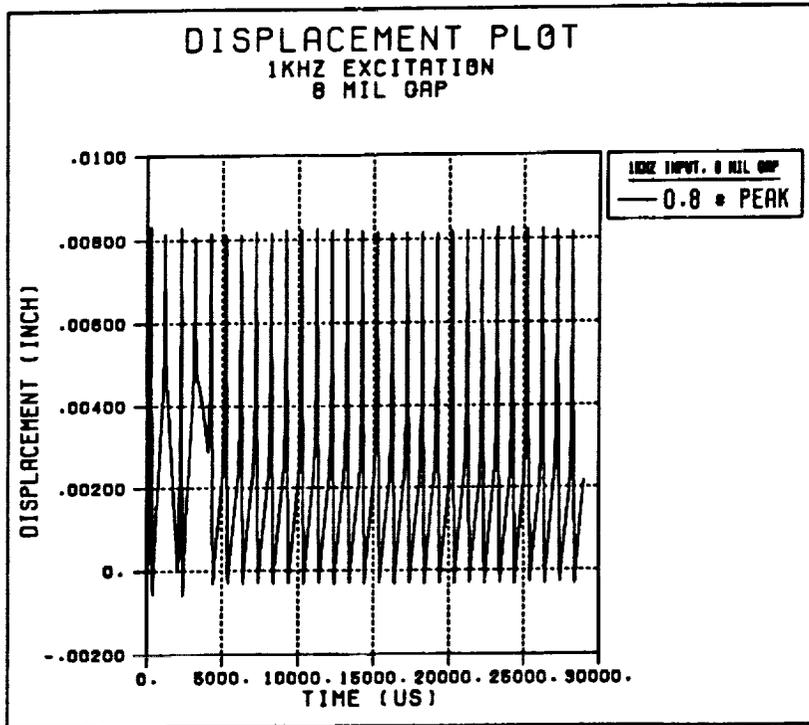


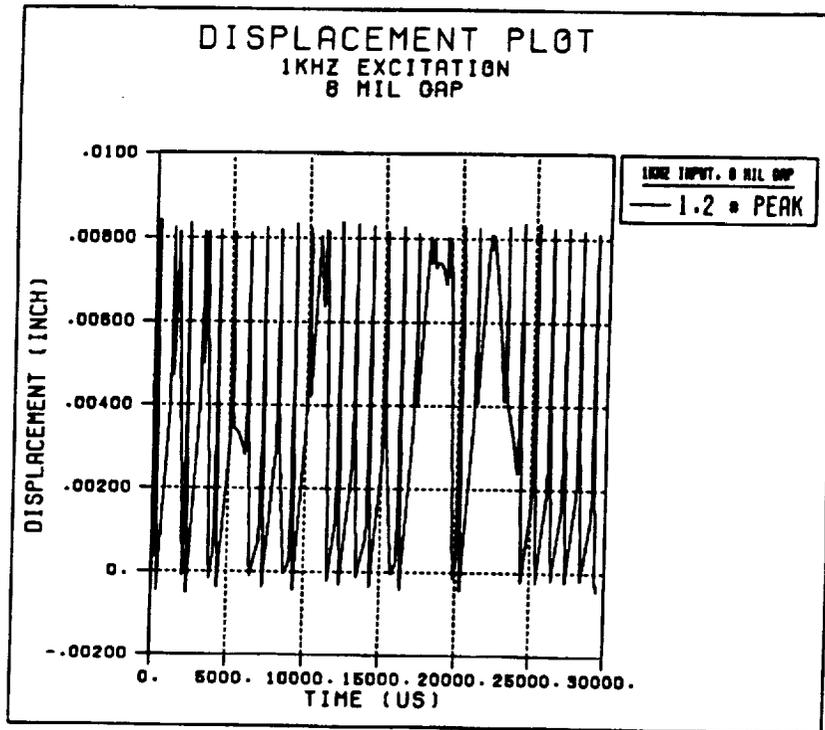
Fig. (2): Restoring force in system



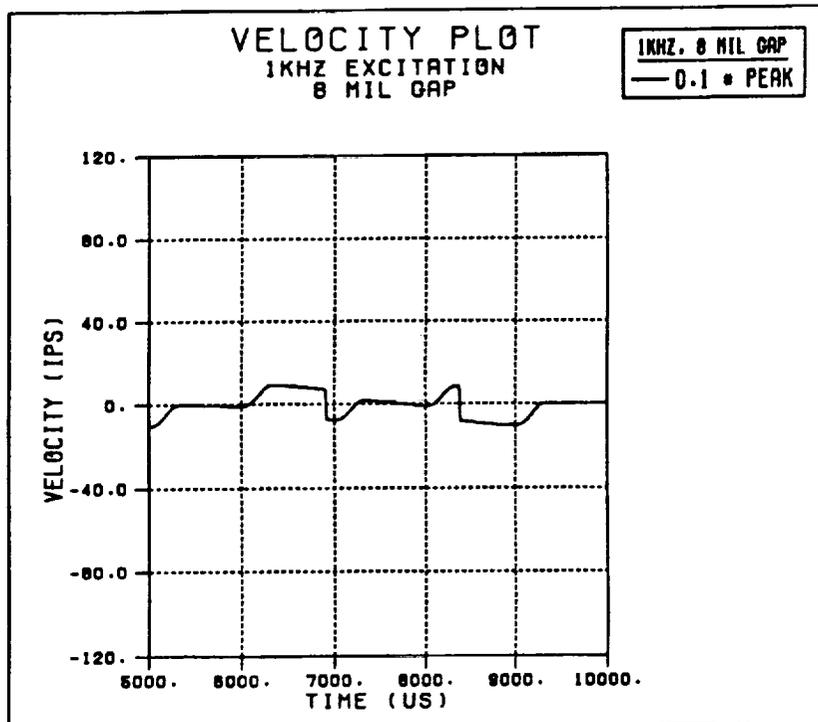
Figs (3-7): Displacement vs. time for armature

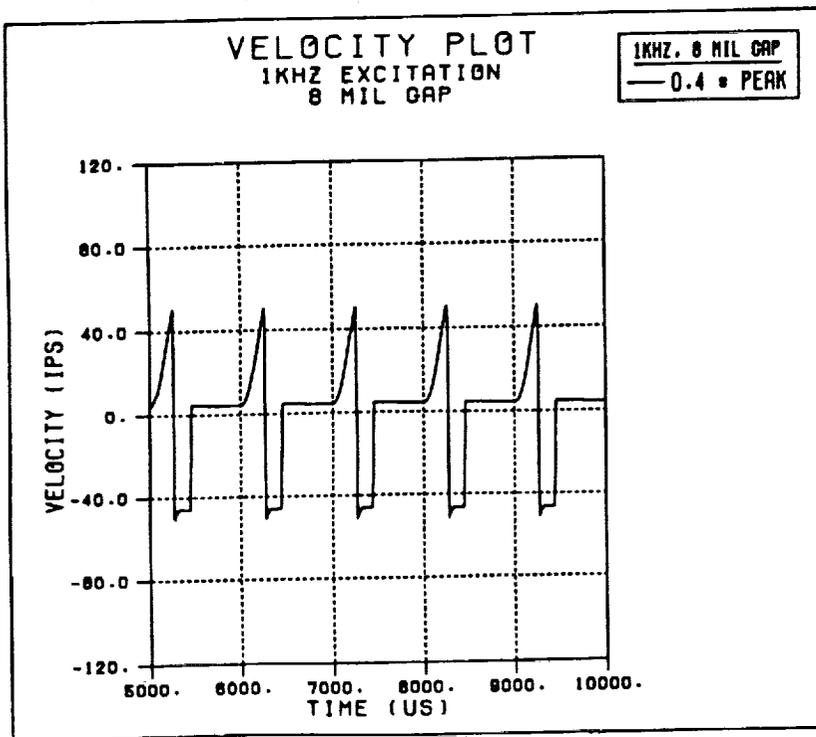
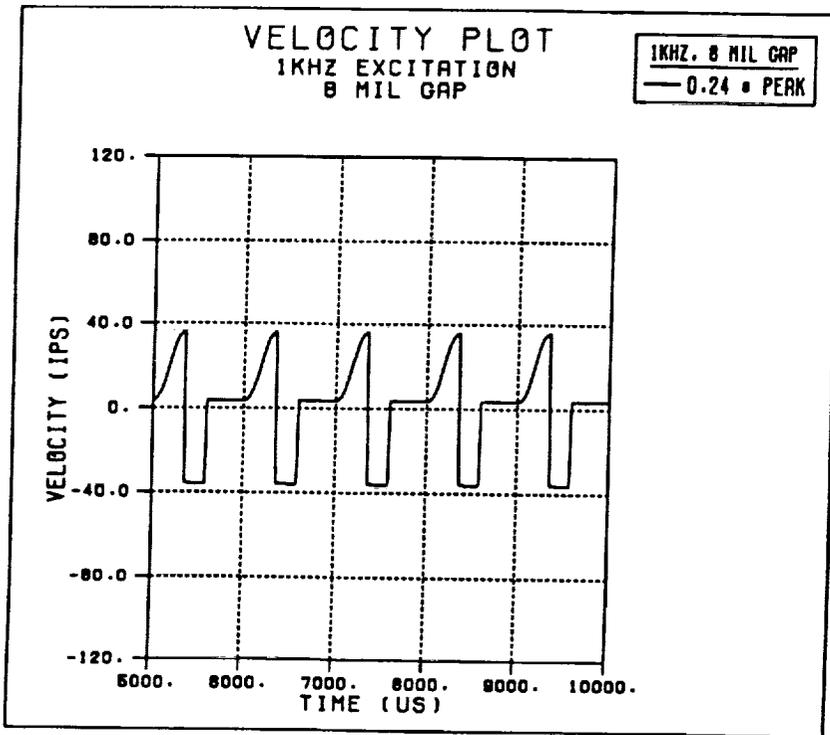


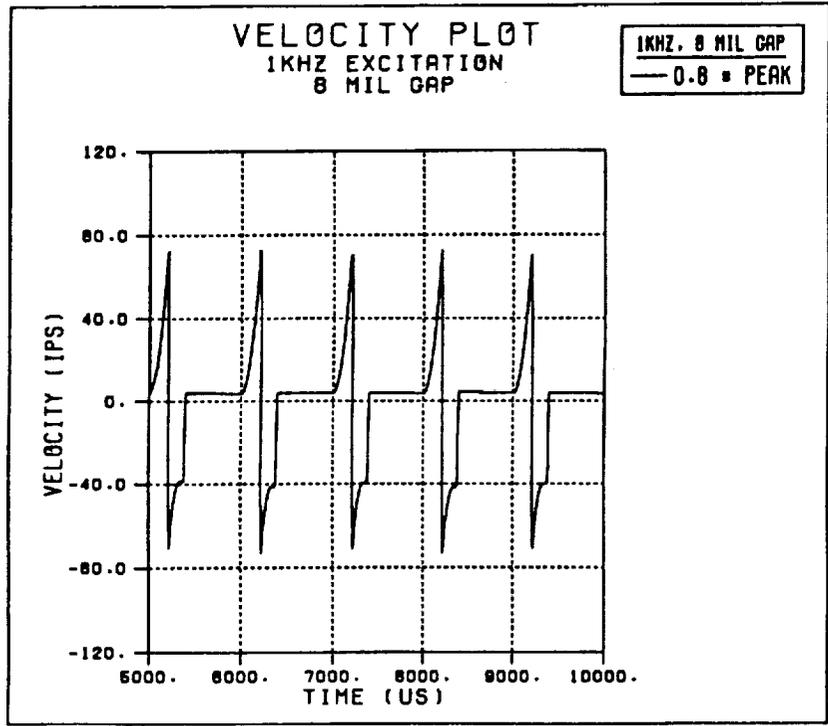
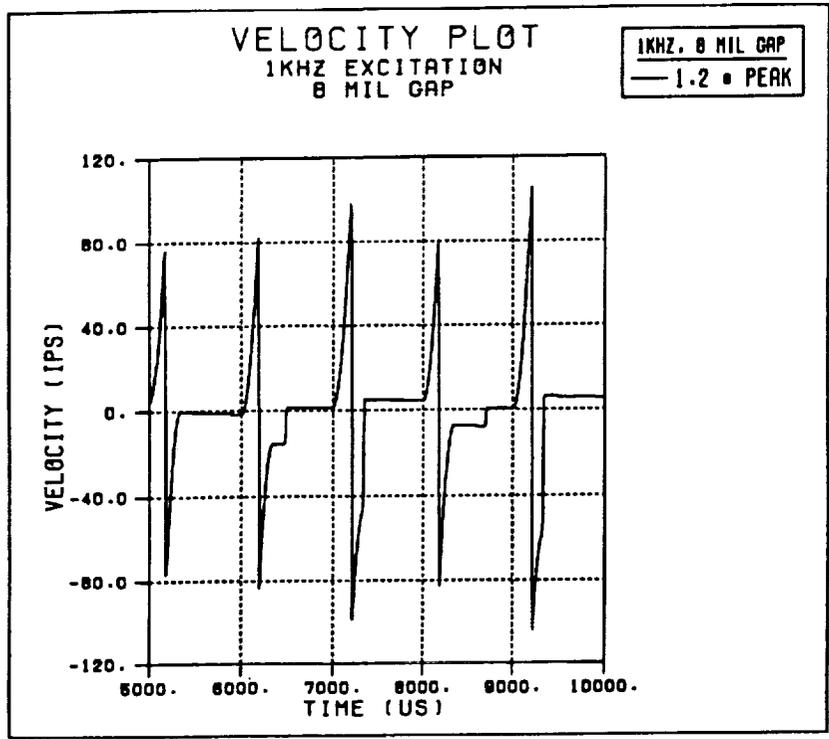




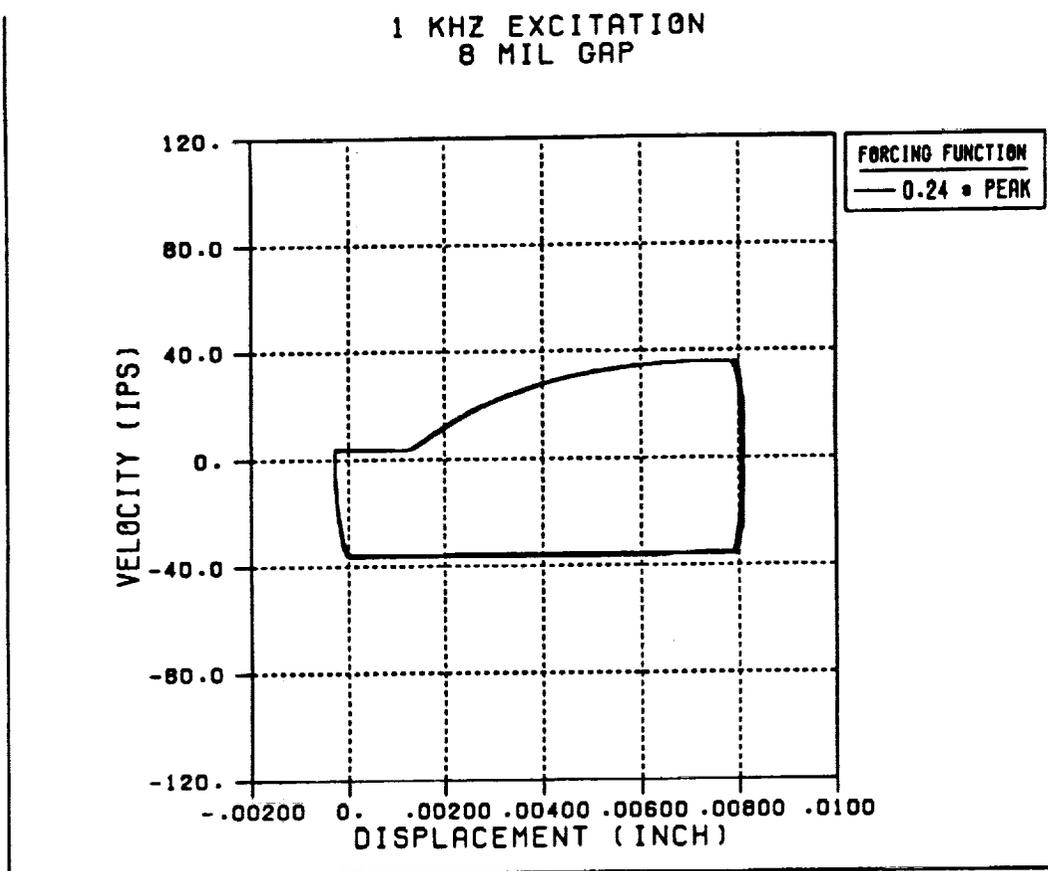
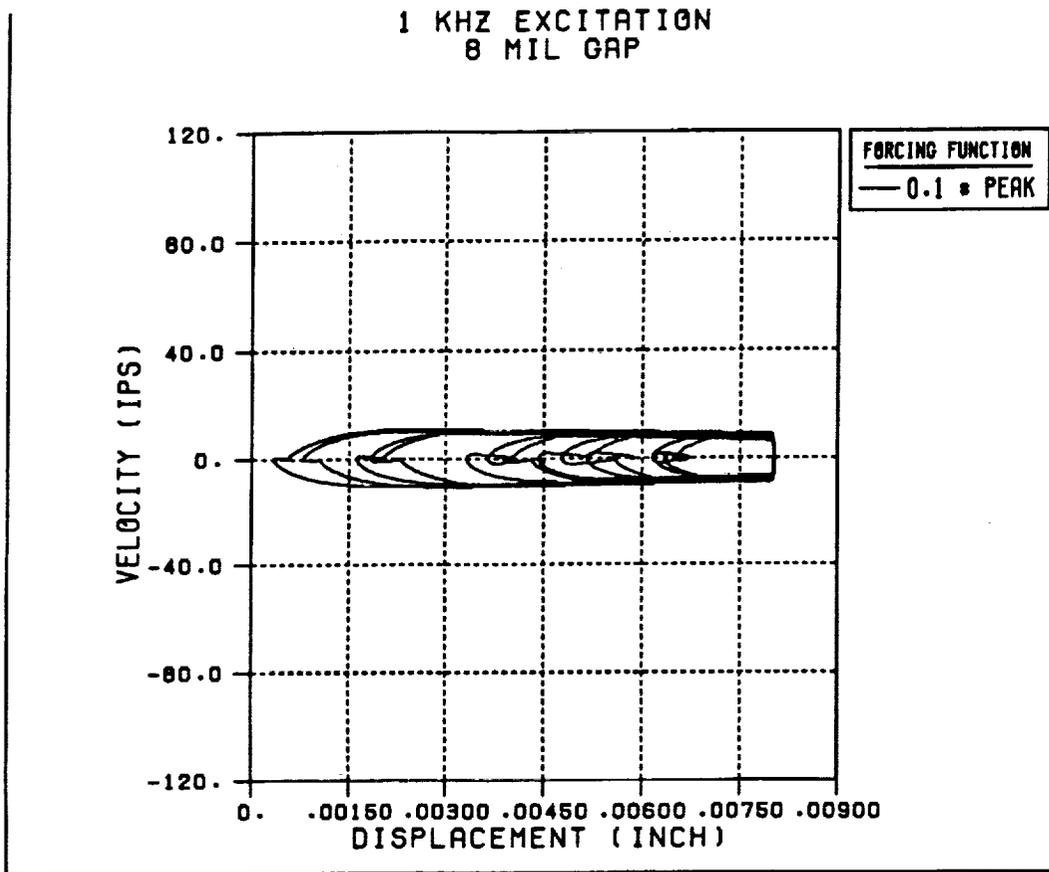
**Figs. (8-12): velocity vs. time for armature**



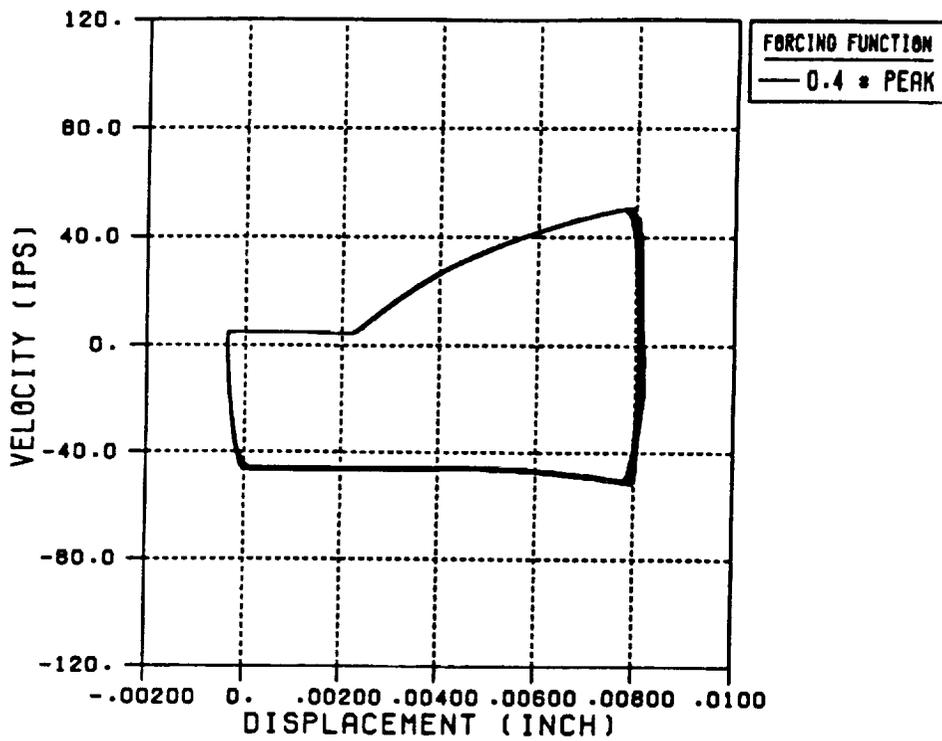




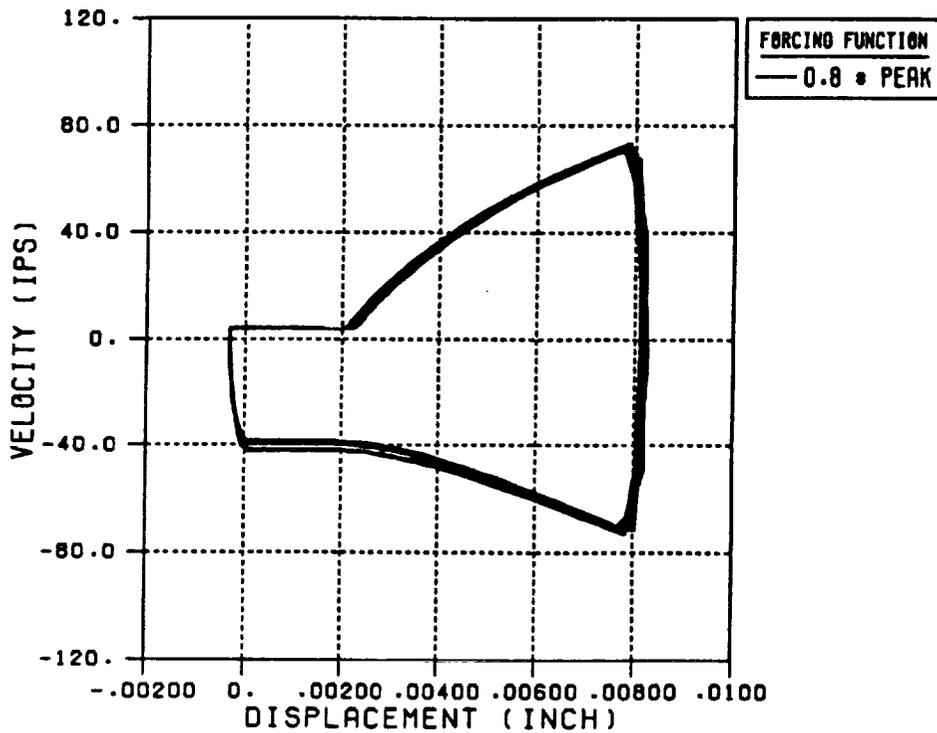
Figs. (13-19): velocity vs. displacement



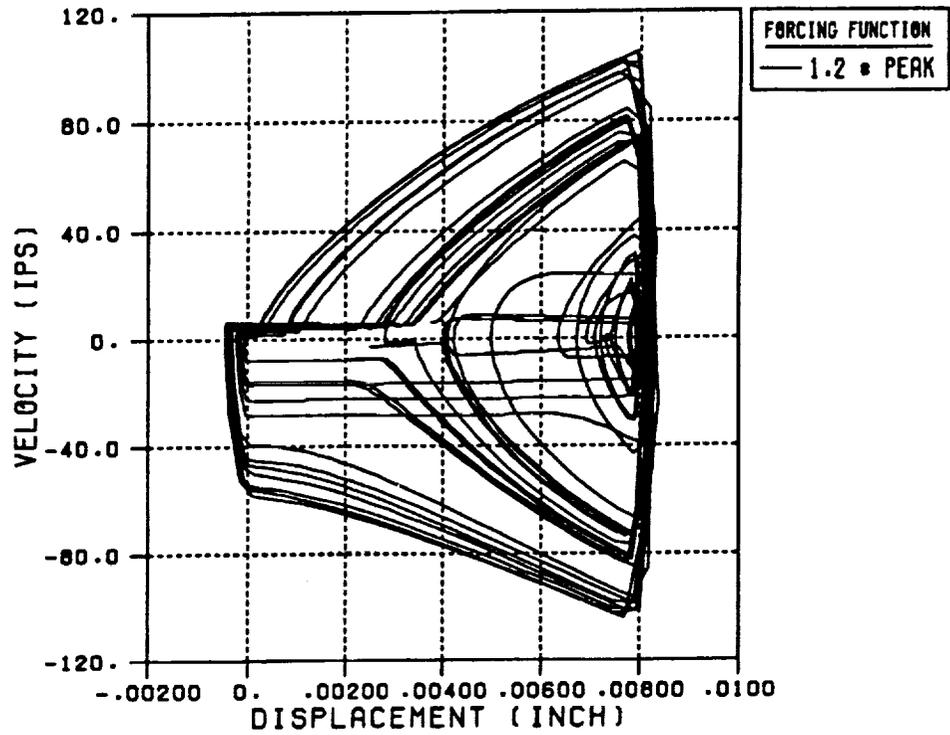
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8 MIL GAP



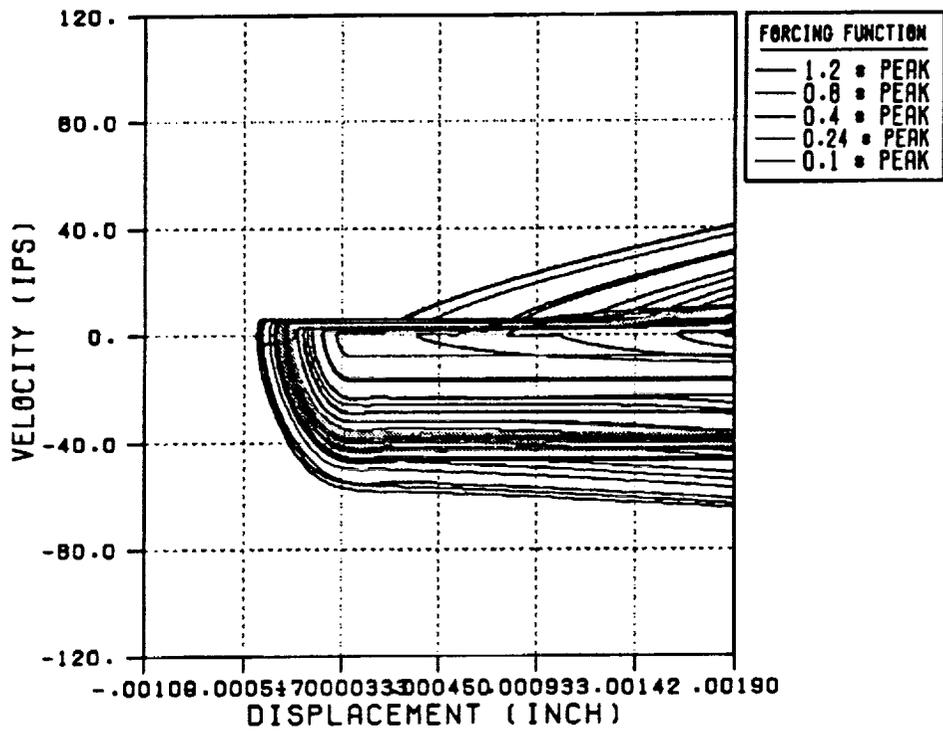
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8 MIL GAP



1 KHZ EXCITATION  
8 MIL GAP



1 KHZ EXCITATION



# 1 KHZ EXCITATION

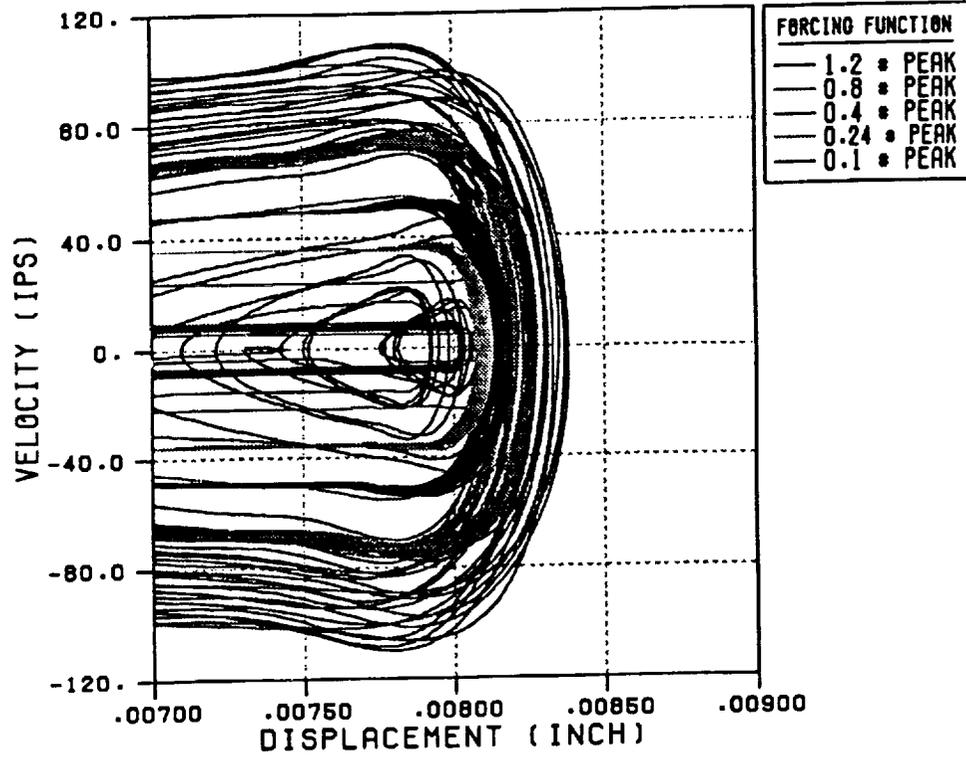
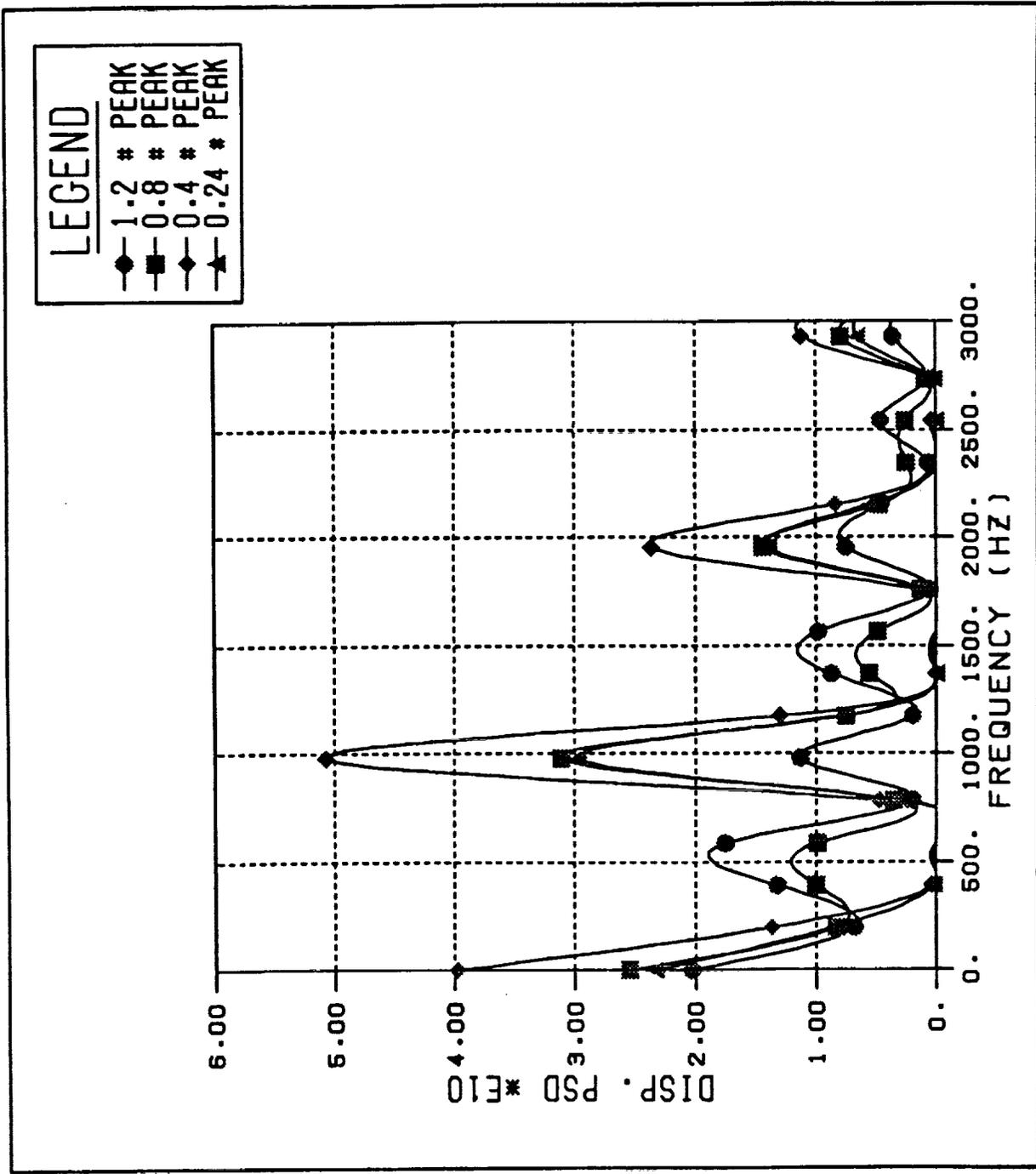
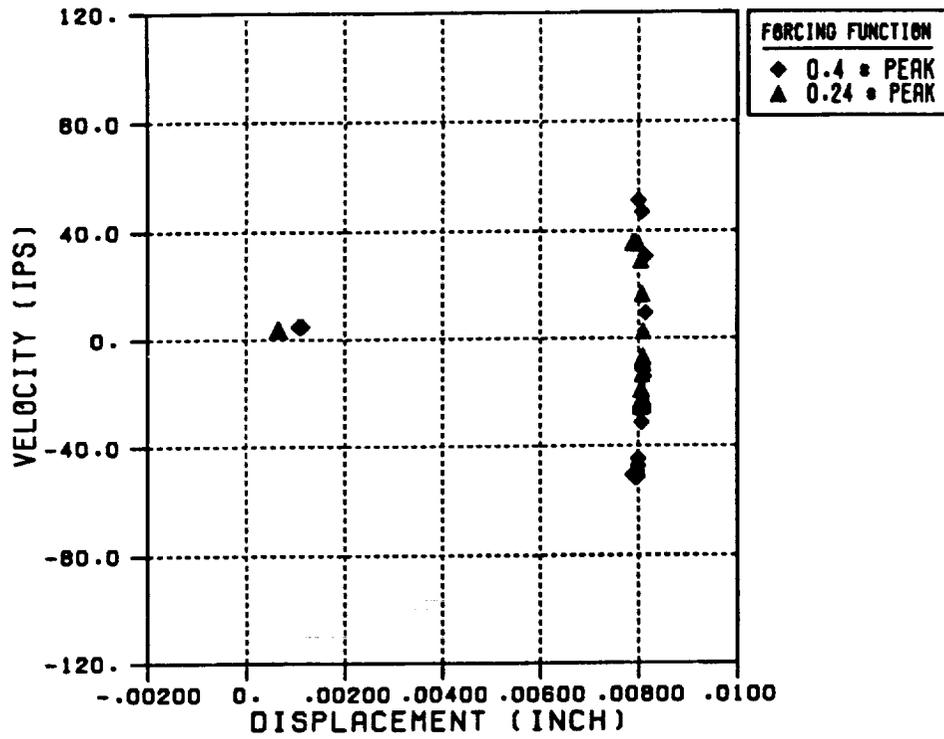


Fig. (20): Displacement Power Spectral Density



Figs (21-22), Poincare plots sampled at 2kHz

POINCARÉ PLOT  
1 KHZ EXCITATION  
180 DEGREE PHASE SEPARATION



POINCARÉ PLOT  
1 KHZ EXCITATION  
180 DEGREE PHASE SHIFT

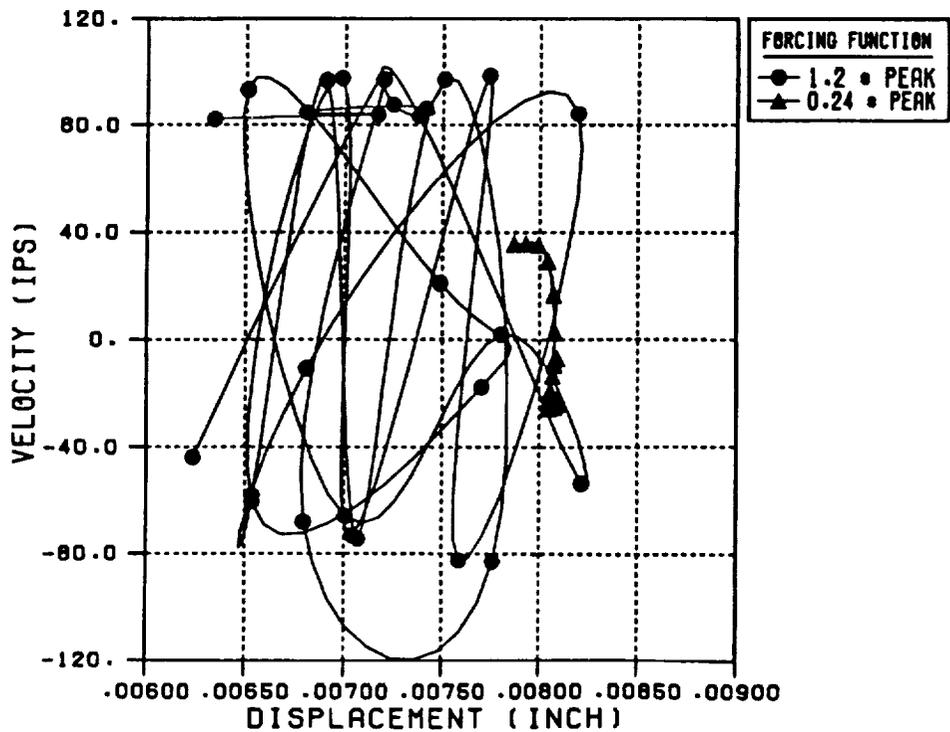


Fig. (23): 3-D plot for 0.8 # peak load

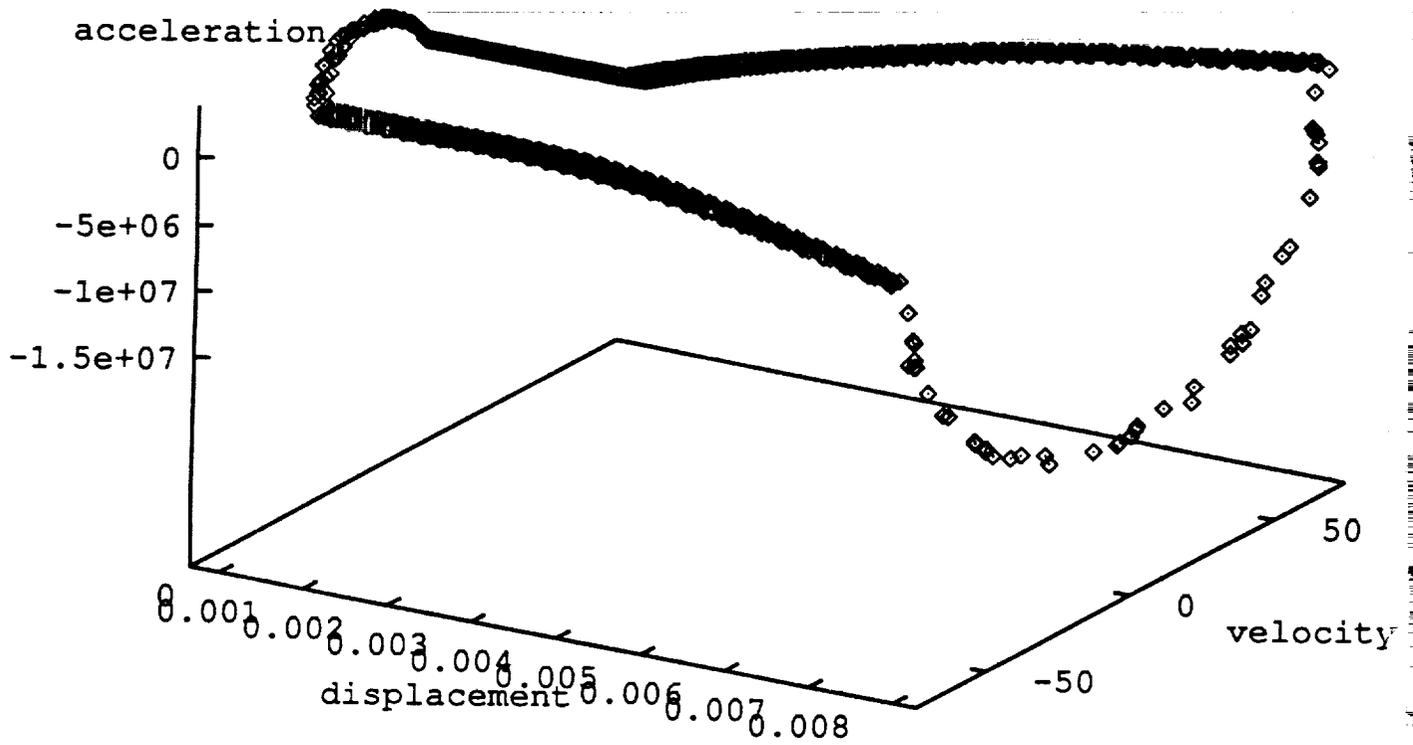


Fig. (24): 3-D plot for 1.2 # peak load

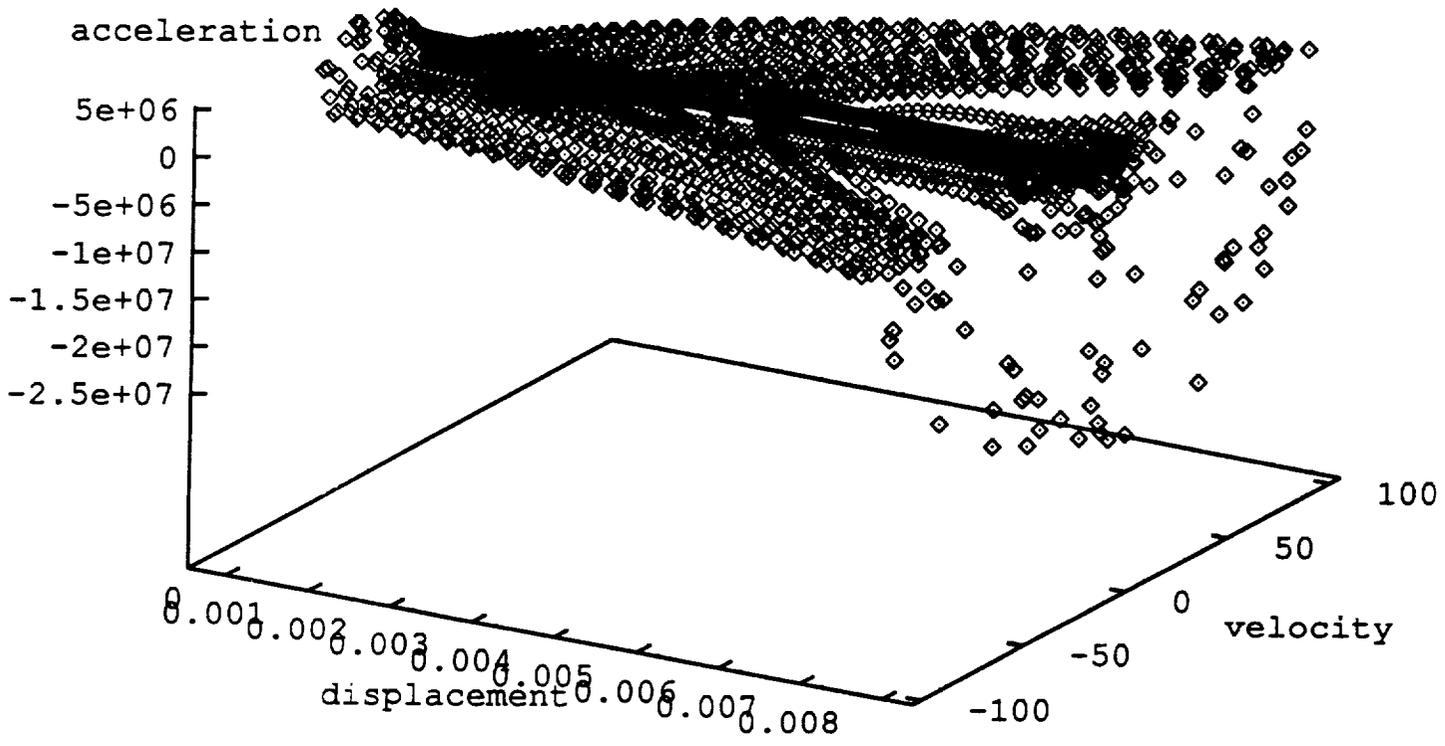


Fig. (25): displacement plots for different linear spring constants

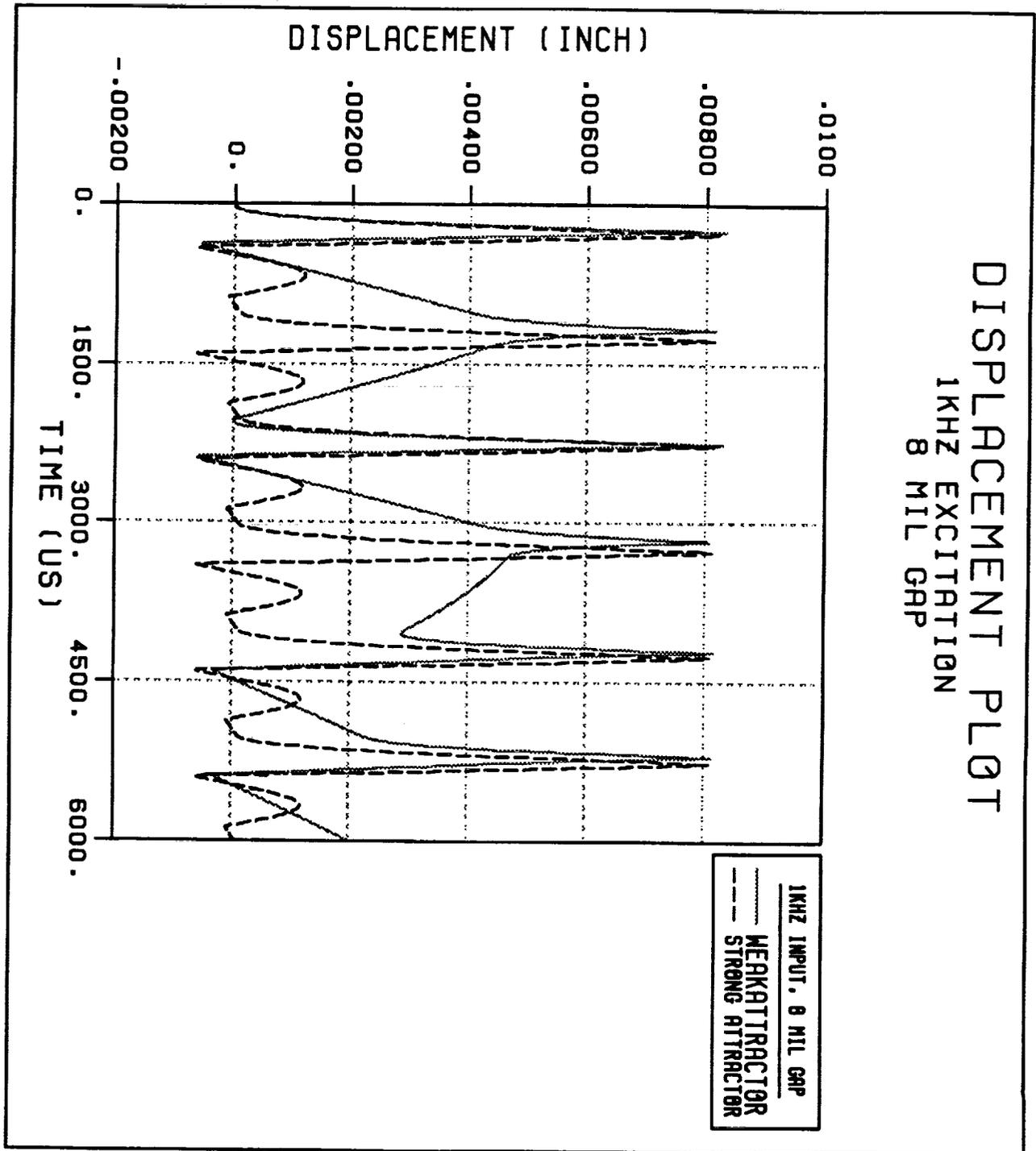


Fig. (26): Phase plots for different linear spring rates

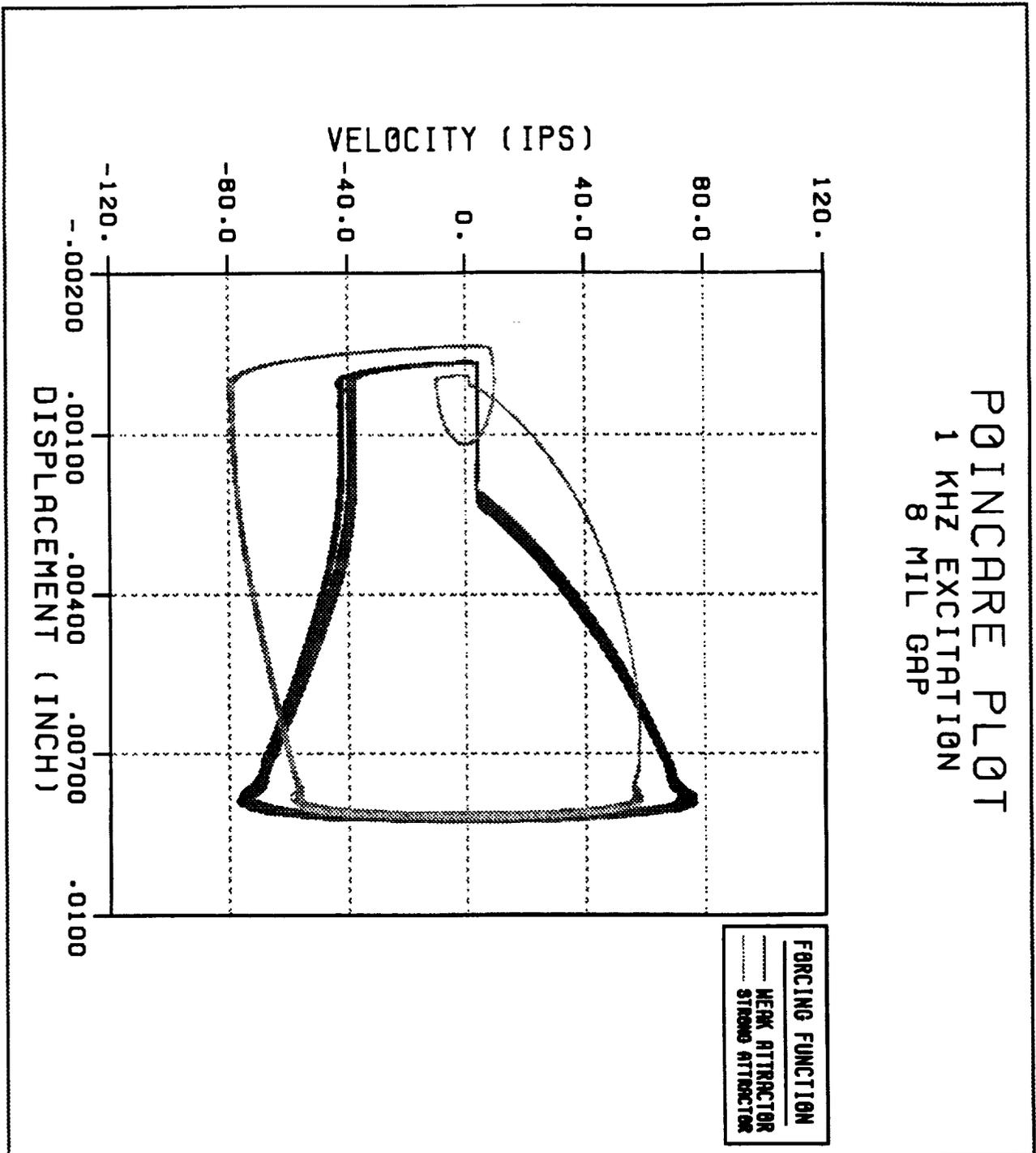


Fig. (27): phase plot at three different forcing freq.s

